Physics 1120

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Contents

1 Charges

Charges come in two types, which we call positive and negative. An atom itself consists of a positively charged nucleus surrounded by a field of negatively charged electrons.

1.1 Terms

Conductor $\rightarrow$ Object in which electrons are not in a fixed position, and are free to move.
Insulator $\rightarrow$ Object in which electrons are in a fixed position, and not free to move.

1.2 Electric Charge

1. Two types of charges: + (positive) and - (negative)
   (a) Protons: +
   (b) Electrons: -
2. Like charges repel, unlike charges attract
3. Charge of an electron is $e = 1.6 \times 10^{-19} C$ where C is a Coulomb
4. Charge is conserved, akin to energy, momentum, etc.

Electrons can move within an insulator, but also rotate within either an insulator or conductor.

1.3 Coulomb's Law

Unlike charges attract while like charges repel. Coulomb's Law (1) models this behavior.
Equation (1) describes the force between two charged particles:

$$\vec{\mathbf{F}}_{1\rightarrow 2} = \frac{K \cdot q_1 \cdot q_2}{r^2} \cdot \hat{r} \tag{1}$$

Where $K = 9.0 \times 10^9 \frac{Nm^2}{C^2}$
This law is similar to the law that describes gravity:

$$\vec{\mathbf{F}}_{grav} = \frac{G \cdot m_1 \cdot m_2}{r^2} \tag{2}$$

1.4 Conservation of Charge

Electric charge is conserved, like matter and energy. The net charge *can not* change.

1.5 Elementary Units

There is an elementary unit of charge that we call e. This is the smallest unit of charge that we can define. There are never fractions of charge.

1.6 Polarization

If we have an uncharged object and place a charged object next to it, despite the lack of charge on the first object, it will move. In conductors this is due to the charges shifting within the object. In insulators this is due to the rotation of each atom and its associated charges.

1.7 Superposition

Net force is equal to the sum of all forces.

$$\vec{\mathbf{F}}_{net} = \sum_i \vec{\mathbf{F}}_i$$

and component force is equal to the sum of those components

$$\vec{\mathbf{F}}_{[x,y],net} = \sum_i \vec{\mathbf{F}}_{[x,y],i}$$

1.8 Dipoles

Any object that has a positive and a negative end. The two ends are separated by a fixed distance. These exist everywhere.

1.9 Continuous Charge Distribution

Although point charges are easy to work with, in reality we need to deal with large objects that have uniform charge. If we are concerned about the charge through a solid, we can describe the behavior of this system in terms of volume charge density ρ. If we care about charges over surfaces or lines, then we use surface charge density ω and the line charge density λ.

In order to estimate the charge from a solid we need to use some calculus:

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}} = \int \frac{k\hat{r}}{r^2} dq \tag{3}$$

2 Electric Field

Surrounding any charge there is a field of energy that we call the electric field. This charge exists whether or not a test charge exists in the field.

The electric field at any point is the force per unit charge that a charge would experience at that point.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_{\text{on q}}}{q} \tag{4}$$

Using Coulomb's Law (1) we can fully express this equation:

$$\vec{\mathbf{E}}_1 = \frac{K \cdot q_1 \cdot q_2}{q_1 \cdot r^2} \cdot \hat{r}$$

Where q_2 is the charge of the object emanating the electric field.

2.1 Matter in Electric Fields

Now we have two different formulas to allow us to model the motion of a particle in charged space, Newton's $F = ma$ and our new description of electric fields $F = qE$ allowing us to get to a new equation to model acceleration in charged space.

$$\vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}} \tag{5}$$

2.2 Electric Field Lines

1. Density of Electric Field Lines $\propto \left|\vec{\mathbf{E}}\right|$
2. Lines start at positive charges and end at negative charges.
3. Number of lines in and out of the object $\propto$ the charge of that object.
4. Direction of $\vec{\mathbf{E}}$ is tangent to lines.

2.3 Conductors and Insulators

Conductors:

- Where ever there is an electric field, charges move. Electrons are free to move around.
- In equilibrium, charges don't move.
- There is no electric field inside a conductor.
- All electric charge (q_{net}) is on the surface.
- Electric field is orthagonal to the surface, parallel to surface vector at that point.

Insulators:

- Can acquire charge through dipole movement or polarization.

3 Gauss' Law

3.1 Electric Flux

In its essence, Electric Flux defines how an electric field passes through a surface. The higher the flux, the more electric field lines are passing through the surface. While flow describes motion across a surface, flux describes motion through it. We can mathematically define flux with equation (6) where $\vec{\mathbf{E}}$ is the electric field and A is the area of the surface.

$$\Phi = \oint \vec{\mathbf{E}} \cdot dA \tag{6}$$

Another method to calculate Electric Flux is a dot product between the two vectors, that of the plane as well as the surface vector as is demonstrated in equation (7).

$$\vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = \left|\vec{\mathbf{E}}\right| \cdot \left|\vec{\mathbf{A}}\right| \cdot \cos(\theta) \tag{7}$$

3.2 Gauss's Law

Essentially Gauss's Law (8) states that the total electric flux is equal to the total enclosed charge divided by a constant.

$$\begin{gathered}\oint \vec{\mathbf{E}} dA = \frac{Q_{\text{enc}}}{\epsilon_0} \\ \text{Where } \epsilon_0 = 8.85 \times 10^{-12} \frac{c^2}{Nm^2} = \frac{1}{4\pi k}\end{gathered} \tag{8}$$

4 Electrostatic Equilibrium

The electric field inside a conductor is in electrostatic equilibrium (No charges moving/exist), however the field at the surface of a conductor can be found with:

$$\vec{\mathbf{E}} = \frac{\sigma}{\epsilon_0} \tag{9}$$

where σ is the surface charge density.

5 Voltage, otherwise known as Electric Potential

Remember from Physics 1110 that work is defined as

$$\begin{gathered} W_F = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \\ \text{For constant force: } \left|\vec{\mathbf{F}}\right| |d\vec{\mathbf{r}}| \cos(\theta) \end{gathered} \tag{10}$$

This equation holds true for electric charges as well.

Right off the bat, it is vital to realize that Electric Potential (Voltage) *is not* the same as Potential Energy.

Electric Potential Difference, ΔV, from point A to B is the potential energy change per unit charge in moving a charge from A to B.

$$\Delta V_{A\to B} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} \tag{11}$$

In the special case of a uniform field, this equation reduces to:

$$\Delta V_{A\to B} = -\vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{r}} \tag{12}$$

We can use Voltage to determine the change in potential energy, using the following formula:

$$\Delta PE = q\Delta V$$

Every charged particle generates an electric field, therefore every charged particle has an electric potential field. Near a positively charged particle, the voltage is high. A way to think of these is to imagine a series of hills and pits, where positive charges create "hills", while negative charges create "pits."

Potential difference is important, and it is measured in volts.

In small systems we have to use the electronvolt (eV) defined as the energy gained by a particle carrying an elementary charge when it moves through a potential difference of one volt. Note, eV is not an SI unit.

If we define a zero potential at $x = \infty$, we can use it to define the potential at any given point in relation to the zero with the following equation.

$$V(r) = \frac{kq}{r} \tag{13}$$

The following equation is an adaption of the Work-Energy Theorem to describe charged particles.

$$\begin{aligned} \Delta U \equiv W_{ext} &= -W_{field} \\ W &= -qV \end{aligned} \tag{14}$$

Since it takes no work to move orthogonal to an electric field, we can define equipotential lines at the points where the change in potential is zero. Think of these as three dimensional shapes, and the equipotentials as contour lines.

6 Electrostatic Energy

Electrostatic energy is defined as the work done to bring a charge to a certain configuration from infinity.

We can apply the principle of superposition to find the sum of these values.

7 Capacitors

Capacitors are pairs of electrical conductors with equal but opposite charges.

The easiest one to work with is the parallel plate configuration.

We previously determined that the charge at the surface of a conductor is $\vec{\mathbf{E}} = \frac{\sigma}{\epsilon_0}$, therefore the charge between the plates is:

$$\vec{\mathbf{E}} = \frac{Q}{\epsilon_0 \vec{\mathbf{A}}} \tag{15}$$

and the potential difference is:

$$V = \vec{\mathbf{E}} d = \frac{Qd}{\epsilon_0 \vec{\mathbf{A}}} \tag{16}$$

7.1 Capacitance

We can define the capacitance of a configuration of two conductors to be:

$$C = \frac{Q}{V} \tag{17}$$

where

$$V = \frac{Qd}{\epsilon_0 \vec{\mathbf{A}}} \therefore C = \frac{\epsilon_0 \vec{\mathbf{A}}}{d} \tag{18}$$

The units of capacitance are in $\frac{C}{V}$, which have their own special name of farads.

7.2 Energy Storage in Capacitors

We can calculate the amount of energy stored in a capacitor which is proportional to the charge and voltage put in.

$$W = \int dw = \int_0^V CV dV = \boxed{\frac{1}{2} CV^2} \tag{19}$$

The energy density of a capacitor is:

$$u_{\vec{\mathbf{E}}} = \frac{U}{V} = \frac{1}{2} \epsilon_0 \vec{\mathbf{E}}^2 \tag{20}$$

To account for the material in between the two plates, we can include the diaelectric constant to get the end capacitance:

$$C = \frac{\kappa \epsilon_0 A}{d} \tag{21}$$

Where κ is called the diaelectric constant and is dependent on the surface. $\kappa_{\text{air}} = 1$

7.3 Parallel Capacitors

Two capacitors in parallel have the same potential difference (voltage), and their charges can be added.

$$\sum Q = Q_1 + Q_2 + \cdots + Q_n$$

7.4 Capacitors in Series

Two capacitors in series have the same charge (Q), and their capacitance is determined the following method.

$$C = \frac{C_1 \cdot C_2 \cdots C_n}{C_1 + C_2 + \cdots + C_n}$$

8 Current

Now we are no longer dealing with charges in static equilibrium. Current can be defined as $I = \frac{dQ}{dt}$ which is the amount of charge crossing an area over dt. The units of charge are Ampéres, or Amps, which are $1amp = 1\frac{coulomb}{sec}$.

- **Q:** How much current can kill someone?
- **A:** $0.01A$ across their heart.

At its core, current is the net rate of charge crossing an area which can be expressed with the following formula.

$$I = \frac{dQ}{dt} \tag{22}$$

When dealing with current through a wire, we have a couple more quantities to deal with.

- $n \rightarrow$ Charges per unit volume
- $q \rightarrow$ Each charge magnitude
- $\vec{\mathbf{V}}_d \rightarrow$ Drift velocity vector
- $L \rightarrow$ Length of the wire
- $A \rightarrow$ Area of cross section

We can then use these variables to calculate a more explicit formula for current through a wire.

$$I = \frac{dQ}{dt} = nAq\vec{\mathbf{V}}_d \tag{23}$$

8.1 Current Density (Ohm's Law at the Microscopic Level)

To find the density of the current at a given point, the following formula is used.

$$\vec{\mathbf{J}} = nq\vec{\mathbf{V}}_d \tag{24}$$

Where $n, q,$ and $\vec{\mathbf{V}}_d$ are declared above.
We can also use the formula

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} \tag{25}$$

where σ is the conductivity of the item and $\vec{\mathbf{E}}$ is the electric field.
Note, at these sizes, thermal noise can adversely affect the circuit.

8.2 Macroscopic Ohm's Law

We can use the prior equations to determine Ohm's law at the macroscopic level.

$$\Delta V = IR \tag{26}$$

This is widely applied, however it is not applicable in all circumstances.

8.3 Summary

To summarize the terms established thus far.

Microscopic Term	Macroscopic Term	Relation
Electric Field $\rightarrow \vec{\mathbf{E}}$	Voltage V	$\vec{\mathbf{E}}$ is the electric field at a point. V generalizes and determines over the region.
Current Density $\rightarrow \vec{\mathbf{J}}$	Current I	As above, $\vec{\mathbf{J}}$ is at a point, while I generalizes and determines current through a region.
Ohm's Law $\rightarrow \vec{\mathbf{J}} = \frac{\vec{\mathbf{E}}}{\rho}$	$I = \frac{V}{R}$	And again, this is a generalization of the microscopic viewpoint.

Table 1: Summary of Terms

8.4 Electric Power

We can calculate energy per unit time for electrical circuits.

$$P = IV = I^2R = \frac{V^2}{R} \tag{27}$$

9 Electrical Circuits

Def: *A collection of electrical components connected by conductors*

An electric field will drive a current through a resistor without issue, however in order to get a fixed potential difference (voltage), and therefore an electric field we need a device called a source of Electromotive Force, or emf.

When a source of emf is connected to a circuit, energy will flow from the positive to negative terminals.

An ideal emf will maintain the same voltage across all of its terminals.

The most common source of this emf is a battery.

9.1 Series Resistors

The current through circuit components in series is the same.

$$\begin{aligned} &\text{Let's define Voltage as } \epsilon \\ I &= \frac{\epsilon}{R_1 + R_2} \\ V_1 &= \frac{R_1}{R_1 + R_2}\epsilon \\ V_2 &= \frac{R_2}{R_1 + R_2}\epsilon \\ R_{\text{series}} &= R_1 + R_2 + \cdots + R_n \end{aligned} \tag{28}$$

9.2 Parallel Resistors

The voltage across circuit elements in parallel is the same.

$$I_1 = \frac{\epsilon}{R_1}$$
$$I_2 = \frac{\epsilon}{R_2} \tag{29}$$
$$R_{\text{parallel}} = \frac{R_1 R_2}{R_1 + R_2}$$

9.3 Kirckoff's Laws & Multiloop Circuits

In a circuit, charges gain energy at emfs and lose it at resistors. For a complete circuit the charge sum is zero. These ideas are expressed in Kirchoff's Laws.

1. *Loop Law* $\rightarrow$ The sum of the voltage changes around a closed loop is zero.
2. *Node Law* $\rightarrow$ The sum of the currents at any node is zero.

Where a node is defined as any junction of three or more wires in a circuit.
We have a strategy for analyzing circuits using Kirchoff's laws:

1. Identify all loops and nodes.
2. Label currents that are zero.
3. For all but one node, write equations expressing Kirchoff's Node Law.
4. For independent loops, use Kirchoff's Loop Law.
    - Voltage from positive to negative of a battery is ϵ.
    - Same voltage in reverse is $-\epsilon$.
    - In direction resistance is identified as $+IR$.
    - Same resistance in the opposite direction is $-IR$.

9.4 Electrical Measurement

- Voltmeter $\rightarrow$ Voltage across terminals.
- Ammeter $\rightarrow$ Current
- Ohmeter $\rightarrow$ Resistance

9.5 Capacitors in Circuits

Up to now we've been dealing with very simple circuits that we make assumptions with such as speed of electric current. Capacitors change this because their charge time isn't instant.

The voltage across a capacitor can not change instantly.

We can look at an RC circuit to model this behavior.

10 Magnetism

Magnetism is a curious phenomenon that involves moving charges.

- Magnetic Force is at right angles to velocity of the charge and the magnetic field $\vec{\mathbf{B}}$.
- Magnitude of force is $\propto$ product of charge q, speed v, and field strength B.
- Force is greatest when charge moves at right angles to the field.
  - Zero for parallel
  - In general, force $\propto \sin\theta$

We have previously defined the electric field as $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. We can use this formula and the properties above to determine the formula for magnetic force.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{30}$$

We also know

$$\left|\vec{\mathbf{F}}_B\right| = |q|\, vB\sin\theta$$

Magnetic and electric fields induce different forces, and when a particle undergoes both magnetic force as well as electric force, we call this electromagnetic force.

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

10.1 Charged Particles in Magnetic Fields

Magnetic forces always act perpendicular to to the particles velocity, therefore these forces never change the particle's speed, only the direction, and it does no work.

Because the force is always perpendicular, we know that the particle will be moving in a circle with radius r. To find the radius of this circle, we can combine newton's law and our magnetism equations to get a formula for the radius.

$$r = \frac{mv}{qB} \tag{31}$$

10.1.1 Cyclotron Frequency

So now we ask, what's the period of this charge? We can determine this with the following formula:

$$f = \frac{qB}{2\pi m}$$

10.2 Magnetic Force on a Current

We know that magnetic fields affect moving charges, therefore it should affect charges passing through a wire.

To determine the magnetic force on a current, we apply the following formula.

$$\vec{\mathbf{F}} = I\vec{\mathbf{l}} \times \vec{\mathbf{B}} \tag{32}$$

Where $\vec{\mathbf{l}}$ is a vector whose magnitude equals the length of the wire in the direction of the current, I is the current, and $\vec{\mathbf{B}}$ is the magnetic field.

10.3 Hall Effect

The hall effect is the seperation of charges across a current carrying wire. The potential difference is called the Hall Potential (33).

$$V_H = \frac{IB}{nqt}$$
$$\text{Where } \frac{I}{nq} \text{is called the Hall Coefficient} \tag{33}$$

10.4 Gauss' Law and Ampére's Law

Remember Gauss' Law from before (8)? We can extrapolate and include Ampére's Law which are reputedly very important...

This version of Gauss Law dictates that the magnetic flux of a surface due to a magnetic field is always zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \tag{34}$$

Ampére's Law is another fundamental electromagnetic law that described magnetic field flow. Why this wasn't said in class is beyond me.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \cdot I_{\text{enc}} \tag{35}$$

10.5 Solenoids

A solenoid is a tightly wound coil of wire that creates a magnetic field. For a long solenoid (on thats length is greater than its diameter) we can use Ampére's Law (35) to determine the magnetic field inside the solenoid.

$$\vec{\mathbf{B}} = \mu_0 n I \tag{36}$$

Where n is the number of turns per unit length ($\frac{N}{L}$) and I is the current.

11 Electromagnetic Induction

For an induced EMF, the field isn't localized as with a normal EMF in a battery. Rather is is spread out over the components.

Like electric flux, magnetic flux is the integral of the magnetic field over a surface.

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \tag{37}$$

For a flat surface, this reduces to

$$\Phi = A \cdot B \cdot \cos\theta$$

11.1 Faraday's Law

Faraday's Law helps us establish behavior for electromagnetic interactions.

Theorem 1 (Faraday's Law of Induction)**.** *The induced emf in a circuit is $\propto$ the rate of change of magnetic flux through any surface bounded by that circuit.*

$$\epsilon = -\frac{d\Phi_B}{dt} \tag{38}$$

Where ϵ is the induced emf and Φ_B is the magnetic flux.

There is an EMF induced in any loop that is equal to the negative of the change in flux with respect to time.

$$\begin{aligned} \epsilon &= -N\frac{d\Phi_B}{dt} \\ \Phi_B &= \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \end{aligned} \tag{39}$$

Where N is the number of loops.

11.2 Lenz' Law

Theorem 2 (Lenz' Law)**.** *The direction of an induced electromagnetic field (EMF) or current is such that the $\vec{\mathbf{B}}$ field created by the induced current opposes change in the magnetic flux that created it.*

This law helps us determine magnetic field interactions when dealing with magnetic flux.

11.3 Inductors

Inductors are devices that get energy from a magnetic field. We have a couple of equations to describe them.

We can first define self-inductance as the ratio of magnetic flux through the inductor to current in the inductor. (The units of self-inductance are Henries)

$$L = \frac{\Phi_B}{I} \tag{40}$$

To find the voltage and current through an inductor as a function of time we can use Faraday's Law from before.

$$\epsilon_L = -L\frac{dE}{dt} = -\epsilon_0 e^{\frac{-Rt}{L}}$$

Which gives us current:

$$I = \frac{\epsilon_0}{R}\left(I - e^{\frac{-Rt}{L}}\right)$$

We can also calculate the energy stored (U) as well as energy density (U_m).

$$\begin{aligned} U &= \frac{1}{2}LI^2 \\ U_m = \frac{U}{Volume} &= \frac{1}{2\mu_0}B^2 \end{aligned}$$

12 Electromagnetism

As of now we have 4 laws of electromagnetism which are detailed in Table (2).

12.1 Electromagnetic Waves

Between Faraday's and Ampere's Laws, we get the notion of a self-propagating electromagnetic wave.

To describe these fields mathematically, we (of course) turn to the sine wave.

$$\begin{cases} \vec{\mathbf{E}}(x,t) = E_p \sin(kx - \omega t)\hat{\jmath} \\ \vec{\mathbf{B}}(x,t) = B_p \sin(kx - \omega t)\hat{k} \end{cases} \tag{41}$$

The speed of this wave is found with $\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ while the wave's amplitude can be found with $E = cB$ where c is the speed of light.

Law	Mathematical Law	What it tells us
Gauss for $\vec{\mathbf{E}}$	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$	How charges produce electric fields. Field lines begin and end on charges.
Gauss for $\vec{\mathbf{B}}$	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	No magnetic charge. Field lines neither begin nor end.
Faraday	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{d\Phi_B}{dt}$	Changing magnetic flux changes electric fields.
Ampere (Steady Currents)	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = \mu_0 I$	Electric field produces a magnetic field.
Ampere With Maxwell's Modification	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$	Electric current and changing electric flux produce a magnetic field.

Table 2: Table of Electromagnetic Laws

12.1.1 Polarization

Polarization specifies direction of electric field, and as the name indicates, polarized light only occurs along one axis. Light from natural sources are for the most part unpolarized (waves occur along every axis orthogonal to the axis of travel).

Nonpolarized light becomes polarized after going through a polarized filter.

The Law of Malus describes how much light gets through the filter.

$$S = S_0 \cos^2(\theta)$$

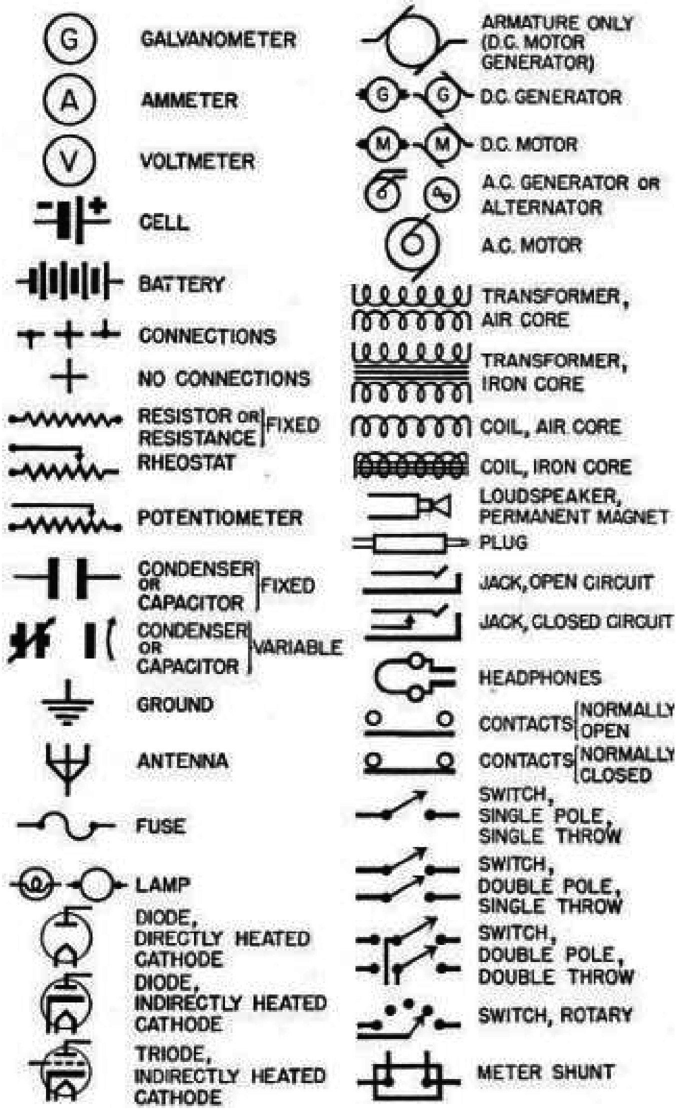

Figure 1: Schematic Diagram Symbols