Physics 1110

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1 1D Motion

We can determine the path of a projectile in one dimension using the below formulas. To determine two dimensional motion we use the same formulas and just determine x and y separately.

$v = \frac{\Delta x}{\Delta t}$, and $a = \frac{\Delta v}{\Delta t}$

$v_f = v_0 + a \cdot t$

$x_f = x_0 + v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$

$v_f^2 = v_0^2 + 2 \cdot a(x_f - x_0)$

To generalize, we can find the parametric path of a given projectile: $\langle v_{0x} \cdot t, v_{0y} \cdot t + \frac{1}{2} \cdot -9.81 \cdot t^2 \rangle$

2 Newton's Laws

We can use Newton's Laws to describe all physical interactions. These laws are as follows:

1. If $F_{net} = 0$; then $\vec{v} =$ constant. In other words, if no force is affecting an object, the object undergoes no acceleration.
2. $F_{net} = m \cdot a$ e.g., net force on an object is equal to the mass times the acceleration.
3. $\vec{f}_{ab} = -\vec{f}_{ba}$ e.g., for every action there is an equal and opposite reaction.

3 Friction

We can calculate the force of friction on an object sliding or rolling over a surface. This force is proportional to the normal force on the object.

Sliding Friction: $\vec{f} = \mu_k \cdot N$

Static Friction: $0 < \vec{f} < \vec{f}_{max} = \mu_k \cdot N$

Sliding friction is the constant force that always is in effect when an object is sliding across a surface. It is dependent on the weight of the object as well as the materials of both the object as well as the surface. Static friction on the other hand is the force that opposes motion from rest. In other words, in order to initiate motion in an object, you have to exert a force greater than the force of static friction.

4 Work

Work can be described as how much energy is expended in how much time.

Work done by a force: $W_f = \vec{f} \cdot \Delta r = \vec{f} \cdot d = \int_a^b \vec{f} dr$

Work-Energy Principle: $W_{net} = \Delta KE$

Work due to a force is equal to:

$$Work = |A|\,|B|\cos(\theta)$$

Where

- $|A|$ is force
- $|B|$ is direction
- θ is the angle at which the force is applied

5 Energy

We can calculate energy of a system and then use these formulas to calculate other unknowns. Every system will have a total amount of energy, and since energy is conserved, the total energy is constant.

There are two types of energy, potential energy and kinetic energy. Potential energy is stored energy, in other words, it is how much energy the system has within it, even though it may seem to be at rest. On the other hand kinetic energy is energy through motion - this type of energy is dependent on how much motion is occurring within the system.

Potential Energy by:

- a conservative force: $\Delta PE = -W_F$
- gravity (approx): mgh
- elastic: $\frac{1}{2}kx^2$
- gravity: $\frac{-Gm_1m_2}{r}$

If there's no friction involved, mechanical energy: $E_{mech} = KE + PE = const.$ Otherwise $E_{mech} = KE + PE + E_{thermal} = const.$

Power (P) is defined as $P = \frac{\Delta w}{\Delta t} \rightarrow \frac{Joules}{sec} \rightarrow (Watts)$.

6 Gravity

We have several formulas to model gravity, both its force as well as the required escape velocity from a planet.

Newton's Universal Law of Gravity:

$$F_{grav} = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.673 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \rightarrow N(\frac{m}{kg})^2$.

We can also find acceleration due to gravity:

$$g = \frac{Gm_1}{r^2}$$

Escape velocity on a planet: $\sqrt{\frac{2Gm}{r}}$

6.1 Kepler's Laws

Johannes Kepler was able to accurately model the motion of celestial bodies using Newton's gravitational laws. This model is a set of three laws that are known as Kepler's Laws.

1. Every planet's orbit is an ellipse with a sun at a focal point.
2. Line from sun to planet sweeps out equal areas at equal times.
3. Period and average sun/planet distance r. $\frac{T^2}{r^3} = const =$ same for all planets.

There is a special case, that of circular orbits:

$$v_{\text{circular orbit}} = \sqrt{\frac{Gm_1}{r}}; \frac{T^2}{r^3} = \frac{4\pi^2}{Gm_1}$$

Velocity of a projectile in orbit:

- $V < escape \rightarrow ellipse$ and $E_{tot} < 0$
- $V > escape \rightarrow hyperbola$ and $E_{tot} > 0$
- $V = escape \rightarrow parabola$ and $E_{tot} = 0$

7 Linear Momentum

Momentum is a measure of how much inertia the system possesses. It is *always* conserved.

We can find the amount of linear momentum a system has using the following formulas:

$$\vec{p} = m \cdot \vec{v}$$
$$\vec{p}_{tot} = \Sigma_i \vec{p}_i$$
$$\text{Units: } kg \cdot \frac{m}{s}$$

Conservation of Momentum: For isolated systems, $\vec{p}_{tot} =$constant

Redefinition of Newton's second Law: $\vec{f}_{net} = m\vec{a} = \frac{d\vec{p}}{dt} = \Delta\vec{p} = \vec{f}_{net,avg} \cdot \Delta t$

Momentum is *always* conserved

Why momentum is conserved? Equal and opposite forces. When momentum is changed in one object, the other's will be the negative, therefore momentum is conserved.

If net force acting on the object is zero, then change in momentum is zero.

7.1 Collisions

We can use momentum to determine how objects in a system will react when in collisions.

- Elastic is when ΣKE is conserved
- Inelastic is when ΣKE is not conserved.
- In a perfectly inelastic collision, objects collide and stick.

Momentum is always conserved, however x and y are conserved separately.

7.2 Impulse

Impulse can be defined as the amount of force times the amount of time the force was applied. Therefore the smaller the impulse, either the less time or the less force, while the larger the impulse, either the more time or more force.

$\tau = F_{net,avg.} \cdot \Delta t = \int \vec{F_{net}} ds$

7.3 2D Momentum

When looking at momentum in two dimensions we have to determine momentum in the x direction and y direction separately.

$\vec{p}_{tot} = const = \Sigma_i \vec{p}_i$

$\therefore$

$\vec{p}_{tot,x} = const = \Sigma_i \vec{p}_{i,x}$

$\vec{p}_{tot,y} = const = \Sigma_i \vec{p}_{i,y}$

8 Rotation

We can use several formulas to model rotation motion. To start, the easy formulas:

- Tangential Acceleration: $|\vec{a}| = \frac{v^2}{r}$
- Tangential Speed: $|\vec{v}| = \frac{d}{t} = \frac{2\pi r}{t}$

There are several definitions that also must be known:

- Radians are defined as arc length divided by radius: $\theta = \frac{s}{r}$
- Angular Velocity = $\omega = \frac{d\theta}{dt}$
- Angular Acceleration = $\alpha = \frac{d\omega}{dt}$
- Tangential Velocity = $r\omega$
- Tangential Acceleration = $r\alpha$

As we defined above to two dimensional motion, If acceleration = $\alpha = const$; then we can use redefined versions of the two dimensional formulas to model motion.

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

We can also calculate torque, which is defined as the amount of tangential force applied at a radius r away from the origin:

$$\tau = r \cdot F_{tangential}$$

8.1 Angular Momentum

$\vec{L} = I \cdot \vec{\omega}$

Angular Velocity $\omega = \frac{d\theta}{dt}$ (For Fixed Axis ONLY)

Direction of angular velocity vector is in direction of Axis for unfixed axes.

$\vec{\omega}$ is parallel to axis (Use right-hand rule to determine positive direction)

$|\vec{\omega}| = |\frac{d\theta}{dt}|$

Angular acceleration vector: $\alpha = \frac{d\vec{\omega}}{dt}$

$\boxed{A_{intiial} + \Delta A = A_{final}}$

Torque Vector $\vec{\tau} = \vec{r} \times \vec{F}$ where $\vec{r}$ is radius and $\vec{F}$ is tangential force

Fixed Axis: $\tau_{net} = I \cdot \alpha$

Any Axis: $\tau_{net} = I \cdot \vec{\alpha}$

Angular Momentum, like Linear Momentum, is a vector which we call $\vec{L}$

$\vec{L} = I \cdot \vec{\omega}$ which is roughly equivalent to $\vec{p} = m \cdot \vec{v}$

One way to look at it is a sum… $\vec{L}_{tot} = \Sigma_i \vec{L}_i = \Sigma_i \vec{r}_i \times \vec{p}_i \rightarrow \hat{z}\Sigma_i r_i m_i \omega r_i \rightarrow \hat{z}\Sigma_i m_i r_i^2 \omega$
Why we care: If net torque is 0, angular momentum is conserved. $\tau_{net} = 0;$ then $\vec{L}_{tot} = c$

$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

$|\vec{\omega}| = \frac{d\theta}{dt}$, and $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_{net} = I\vec{\alpha}$ which is like $F = ma$, but only if I is constant

Momentum of a particle is defined as $\vec{L}_{particle} = \vec{r} \times \vec{p}$, which depends on where the origin is arbitrarily placed.

Conservation of angular momentum states that if the $\tau_{net} == 0, \rightarrow \vec{L}_{tot} = const$

Momentum of Inertia for a system can be defined as $I = \Sigma_i m_i r_i^2$

8.2 Rotation Kinetic Energy

$\frac{1}{2} \cdot I \cdot \omega^2$

8.3 Moments of Inertia for some shapes

- Point mass m at distance r from the axis of rotation $\rightarrow I = mr^2$
- Rod of length l and mass m with axis of rotation at the end of the rod $\rightarrow I = \frac{ml^2}{3}$
- Rod of length l and mass m with axis of rotation at the center of the rod $\rightarrow I = \frac{ml^2}{12}$
- Hoop with radius r and mass m with axis of rotation through the center $\rightarrow I = mr^2$
- Disk with radius r and mass m with axis of rotation through the center $\rightarrow I = \frac{1}{2}mr^2$
- Sphere with radius r and mass m with axis of rotation through the center $\rightarrow I = \frac{2}{5}mr^2$
- Shell with radius r and mass m with axis of rotation through the center $\rightarrow I = \frac{2}{3}mr^2$

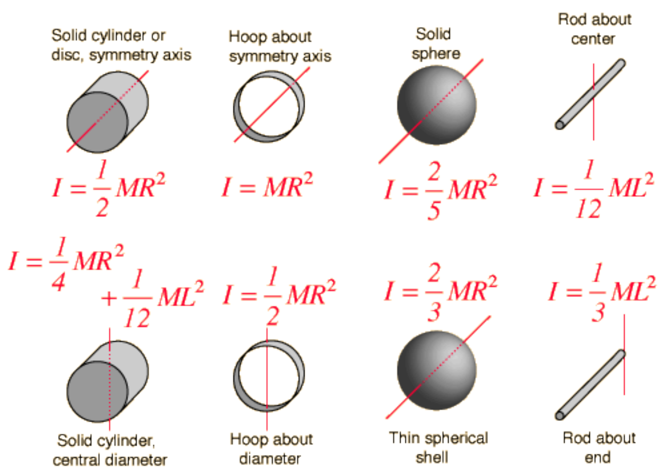

Figure 1: Moments of inertia

8.4 Formulas

Useful listing of formulas. These get confusing, so reference them often.

1. Angular Velocity: $\frac{d\theta}{dt} = \omega$
2. Angular Acceleration: $\alpha = \frac{d\omega}{dt}$

3. Moment of Inertia = $I = \Sigma mr^2$

4. Angular Momentum: $\vec{L} = I \cdot \vec{\omega}$

5. Net Torque: $\tau_{net} = I \cdot \vec{\alpha}$

6. Rotational Kinetic Energy = $KE_{rot} = \frac{1}{2}I\omega^2$

7. Total Kinetic Energy = $KE_{tot} = KE_{linear} + KE_{rotational} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

9 Static Equilibrium

Essentially static equilibrium is when all forces are in balance and no force in dominant.

Static Equilibrium assured if two conditions are met:

1. $\Sigma F_x = 0$, and $\Sigma F_y = 0$
2. $\Sigma_i \tau_i = 0$ about *any* axis

10 Simple Harmonic Motion

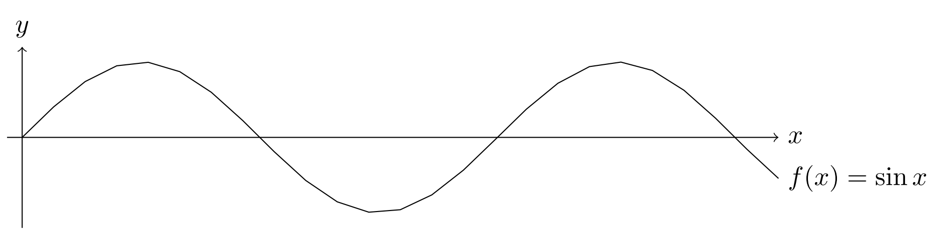

Figure 2: An example of Simple Harmonic Motion: A Sine Curve

$\propto$ is "proportional to"

1. $F_{restore} \propto$ displacement from equilibrium (like $F_{spr} = -k \cdot x$)
2. $PE \propto x^2$ (like $PE_{elas} = \frac{1}{2}k \cdot x^2$)
3. Any time period T does not depend on the amplitude of the motion
4. x, v, a vs time creates a sinusoidal curve

For springs, Hooke's Law is as follows:

$$F_{spring} = -kx$$

and:

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

while simple pendulums can be described with:

$$\omega = \sqrt{\frac{g}{L}}$$

We also have formulas to describe this motion.

$$\begin{cases} x(t) = A\cos(\omega t + \phi) \\ v(t) = -A\omega\sin(\omega t + \phi) \\ a(t) = -A\omega^2\cos(\omega t + \phi) \end{cases}$$

Total energy of the system:

$$E_{tot} = \frac{1}{2}\left(mv^2 + kx^2\right) = \frac{1}{2}\left(kA^2\right) = \frac{1}{2}mv_{max}^2$$

10.1 Differential Equation for Simple Harmonic Motion

So far we've been limited to using $F_{net} = ma$

Now we need to use $\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$

We seek solutions $x = x(t)$ such that $\frac{d^2x}{dt^2} = -\omega^2 x$

Guess and check:

$$\begin{aligned} x = x(t) = Acos(\omega t + \phi) \\ \frac{d^2x}{dt^2} = -\omega^2 \cdot Acos(\omega t + \phi) = -\omega^2 x \\ \therefore \omega = \sqrt{\frac{k}{m}} \end{aligned} \tag{1}$$

10.2 Conservation of Energy for Simple Harmonic Motion

$E_{tot} = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const$ (IF NO FRICTION)

11 Fluids

Pressure is force per area, $p = \frac{F}{A}$. Units are $\frac{Newtons}{meter^2} = Pascal = Pa$.

Mass Density is mass per volume, $\rho = \frac{m}{v}$, and $m = \rho v$.

Pressure due to weight of fluid above: Let's assume we have a bucket with height h, and area of the bottom of the bucket a. The pressure at the bottom is $p = \rho gh$ where g is the force of gravity. Also, the change in pressure due to the change in h is equal to $\Delta p = \rho g \Delta h$. This formula also assumes that the water density is constant throughout the bucket. The pressure calculated above also applies in a lateral direction. Pressure in a fluid is the same in every direction.

The above formula also states that the pressure at any given point relies solely on the depth from the surface.

One atmosphere of pressure is $P_{atm} = 1.01 \times 10^5 Pa$

11.1 The Buoyant Force

Suppose we have a container filled with water. Let's say we have a brick being held in the water. Since the brick takes up a certain volume within the fluid, there is a greater force on the bottom of the object than the top. Therefore the Buoyant force can be defined as the net force acting on an object while the object is in a fluid. If the buoyant force is greater than the force of the object due to gravity, it will float. If not, it will sink.

$F_{buoy} = \Sigma F_{pressure}$

Archimedes is the discoverer of the buoyant force. The upward buoyant force on a submerged object is simply equal to the magnitude of the weight of the displaced fluid.

Archimedes Principle is as follows:

$$\begin{aligned} |\text{Buoyant Force}| &= |\text{Weight of Displaced Fluid}| \\ F_{buoy} &= m_{fluid} \cdot g = \rho \cdot V \cdot g \end{aligned} \tag{2}$$

Where ρ is the density of the mass.

12 Waves

Waves are a self propagating disturbance in a medium. For example, sound is a pressure wave in air.

Waves can be categorized in a couple different ways. One way is transverse vṡlongitudinal. With a transverse wave, the wave is orthogonal to the medium. With a longitudinal wave, it is parallel.

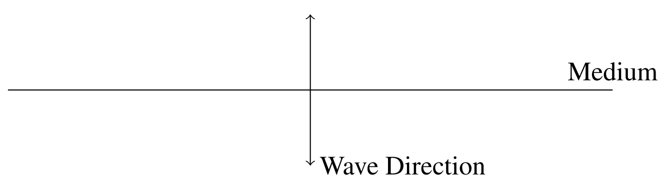

Figure 3: Traverse Wave

Figure 4: Longitudinal Wave

With any medium, if there is just one action that propagates a wave, it is called an impulse wave. If the impulse wave is repeated, it can be sinusoidal.

Sinusoidal traveling waves have a couple different parameters that define them: λ defines the wavelength, and T defines the period. We can relate these in the formula $v = \frac{dist}{time} = \frac{\lambda}{T} = \lambda \cdot f$ where f is $\frac{\text{number of cycles}}{\text{time}} = \frac{1}{T}$. For the most part v is constant, meaning as λ changes, f changes as well.

The speed of the wave depends on the properties of the medium that carries the wave. For example, the speed of sound depends on the air that it's traveling through.

$$v_{\text{sound}} \approx 340m/s, v_{\text{sound}} \propto \sqrt{\frac{\text{Air Temp } T}{\text{Molecule Mass } m}}$$

$$v_{\text{string wave}} = \sqrt{\frac{\text{Tension } T}{\text{Mass/Length } m}}$$

General traveling wave

$$y = f(x,t) = f(x \pm vt) \begin{cases} - \rightarrow \text{right going} \\ + \rightarrow \text{left going} \end{cases}$$

Sinusoidal wave $\rightarrow A \sin\left(2\pi \frac{x}{\lambda}\right)$. To make things easy we will define $k = \frac{2\pi}{\lambda}$, therefore we can rewrite our equation as $y = A\sin(k(x - vt)) = A\sin(kx - \omega t)$ where $\omega = \frac{2\pi}{T}$.

Standing waves exist when there are two sinusoidal traveling waves with the same frequency and amplitude in opposite directions.

Musical tones are often a mixture of frequencies

1. f_1 = fundamental = 1^{st} harmonic
2. $f_2 = 2 \cdot f_1 = 3^{rd}$ harmonic
3. $f_3 = 3 \cdot f_1 = 4^{th}$ harmonic
4. $f_n = n \cdot f_1 = n^{th}$ harmonic

These harmonics are all an increase in frequency. The first harmonic has wavelength $L = \frac{1}{2}\lambda$, and frequency $f_1 = \frac{v}{2L}$. The second has $L = \lambda$ and $f_2 = 2f_1$, etc.

13 Heat and Temperature

Temperature is a measure of energy per atom. The higher the temperature, the more energy the item in question has. There are a couple scales to measure temperature.

- Fahrenheit $\begin{cases} \text{Boiling} = 212°F \\ \text{Freezing} = 32°F \end{cases}$
- Celsius $\begin{cases} \text{Boiling} = 100°C \\ \text{Freezing} = 0°C \end{cases}$
- Kelvin $\begin{cases} \text{Boiling} = 373°K \\ \text{Freezing} = 273°K \\ T_K = T_C + 273 \end{cases}$

Absolute zero is the lowest temperature possible. It indicates zero movement. Absolute zero = $0°K$ = $-273°C$ = $-459°F$.

Heat Q is thermal energy transferred between objects due to temperature differences. We can define a unit of heat to be a calorie which is equal to $4.184J$ which is the amount of energy to heat one gram of water by one degree Celsius. 1 kcal is equal to 1 Calorie (food calorie).

We also have a term called heat capacity. Heat capacity $= \frac{\text{Heat Added}}{\text{Temperature Increase}} = \frac{\Delta Q}{\Delta T}$. Specific heat is also defined as $\frac{\Delta Q}{m \cdot \Delta T} = \frac{\text{Heat Capacity}}{\text{mass}}$. This can be rewritten as

$$\Delta Q = m \cdot c \cdot \Delta T \tag{3}$$

The specific heat of liquid water is $1 \frac{cal}{g \cdot C^\circ}$.

Thermal equilibrium is when all objects have the same temperature.

Latent heat is the energy per mass required to induce a phase change. $L = \frac{\text{energy}}{\text{mass}} = \frac{\Delta Q}{m}$ for a phase change. We can also get $\Delta Q = m \cdot L$.

For H_2O it takes $L_{S \to L} = 80 \frac{cal}{g}$ and $L_{L \to G} = 539 \frac{cal}{g}$.

13.1 Heat Transfer Mechanisms

1. Conduction $\to$ Heat transfer through atomic contact (rather slow)
2. Convection $\to$ Heat transfer by bulk motion of hot matter
3. Radiation $\to$ Heat transfer by light (electromagnetic radiation). Anything with heat is giving off electromagnetic radiation.